(Solns)

Math 2E Quiz 5 Afternoon - April 28th

Please write your name and ID on the front.

Show all of your work, and simplify all your answers. *There is a question on the back side.

1. Sketch the gradient vector field, ∇f, of the following function $f(x, y) = (x + y)^2$ by:

(a) First, find ∇f. When is $\nabla f = (0, 0)$?

2 pts

$\nabla f(x,y) = \langle 2x+2y, 2x+2y \rangle$; $\nabla f = 0$ when $y = -x$.

+1 +1

(b) Since $f \geq 0$, its level surfaces are always non-negative. Solve the xy-level-set-curves from the level sets $f(x, y) \equiv k$, in other words from $(x + y)^2 = k$, for $k = 0, 1, 4, 9$. Note the LHS is squared!

3 pts

$(x+y)^2 = k \Rightarrow (x+y) = \pm\sqrt{k}$, $y = -x \pm \sqrt{k}$.

(-1) if forgot about $-\sqrt{k}$.

When

$k=0$,	$y = -x$	
$k=1$,	$y = -x \pm 1$	+1
$k=4$,	$y = -x \pm 2$	+1
$k=9$,	$y = -x \pm 3$	+1

3 pts

(c) Draw the level set curves with $k = 0, 1, 4, 9$ from (b) on the given plot. Label them, too.

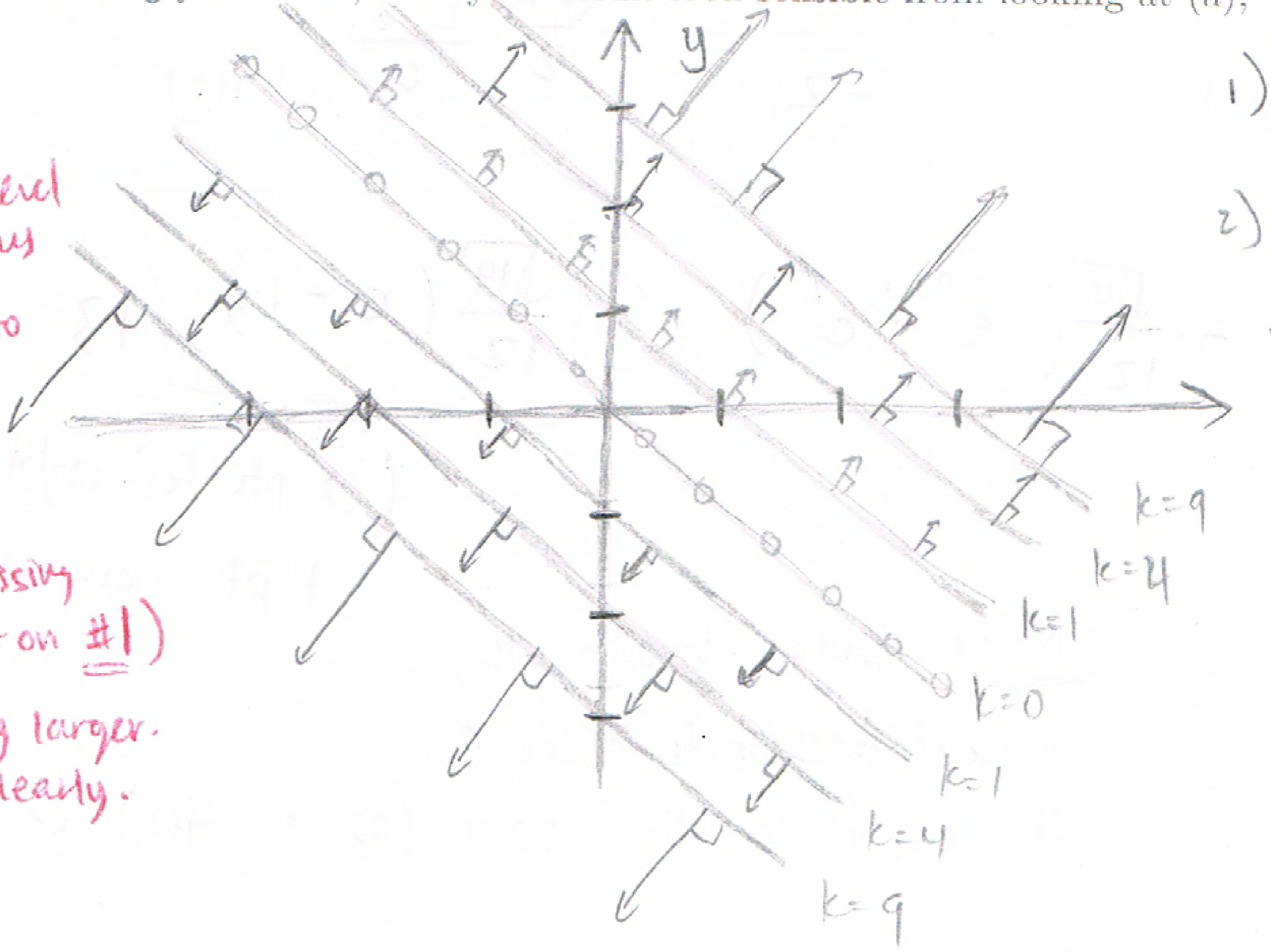
(d) On the same plot, also sketch the gradient vector field ∇f based on your answers from (b) and (c). Use a small "o" along wherever the gradient is zero.

As a means of checking your work, does your result look sensible from looking at (a), ∇f?

+1: level curves

+1: ∇f $\perp$ to level curves

+1: ∇f points to higher k.



1) ∇f $\perp$ to the level sets

2) ∇f points to higher k.

Bonus Pt: (If missing credit on #1)

If ∇f is getting larger. is drawn clearly.

Note: For #3 you could redo your HW problem and just invert the sign, too

3 pts

2. Set up the integral that would evaluate $\int_C \mathbf{F}\cdot d\mathbf{r}$ where $\mathbf{F}(x,y,z) = <\sin(x), \cos(y), xz>$ and the curve C is parameterized by $\mathbf{r}(t) = <t^3, -t^2, t>$ with $0 \le t \le 1$. Don't evaluate it!

- $\vec{r}\,'(t) = <3t^2, -2t, 1>$ +1
- $\vec{F}(\vec{r}(t)) = <\sin t^3, \cos(-t^2), t^3\cdot t> = <\sin t^3, \cos t^2, t^4>$ +1

$$\Rightarrow \int_C \vec{F}\cdot d\vec{r} = \int_0^1 \left(3t^2\sin(t^3)\,dt - 2t\cos(t^2)\,dt + t^4\,dt\right)$$ +1

(9 pts)

3. Compute $\int_C (x+2y)dx + x^2dy$ where C consists of the line segments from (3,0) to (2,1) and from (2,1) to (0,0). (This curve is the backwards orientation from the homework.)

1) From (3,0) to (2,1) : $C_1 \Rightarrow \vec{r}_1(t) = (1-t)<3,0> + t<2,1>$

4 pts +2

$= <3-t, t>, \ (0\le t\le 1)$

(so $dx = -dt$, $dy = dt$)

$$\Rightarrow \int_{C_1} (x+2y)dx + x^2dy = \int_0^1 \underbrace{(3-t+2t)}_{3+t}\cdot(-dt) + (3-t)^2\,dt \ \} +2$$

$$= -\frac{t^2}{2} - 3t\Big|_0^1 - \frac{(3-t)^3}{3}\Big|_0^1 = -\frac{7}{2} - \frac{8}{3} + 9$$ (+1/2) part of Total

2) From (2,1) to (0,0) : $C_2 \Rightarrow \vec{r}_2(t) = (1-t)<2,1> + t<0,0>$

4 pts +2

$= <2-2t, 1-t>, \ (0\le t\le 1)$

(so $dx = -2dt$, $dy = -dt$)

$$\Rightarrow \int_{C_2} (x+2y)\,dx + x^2dy = \int_0^1 \underbrace{(2-2t+2-2t)}_{4(1-t)}\cdot(-2dt) + 4(1-t)^2\cdot(-dt) \ \} +2$$

part of Total (+1/2)

B/c we reversed orientation

1 pt.

$$= -8\left(t - \frac{t^2}{2}\right)\Big|_0^1 + \frac{4(1-t)^3}{3}\Big|_0^1 = -8\left(\tfrac{1}{2}\right) - \frac{4}{3}$$

Total: $\int_C = \int_{C_1} + \int_{C_2} \Rightarrow 9 - \frac{8}{3} - \frac{7}{2} - 4 - \frac{4}{3} = 5 - \cancel{\frac{12}{3}}^{4} - \frac{7}{2} = -\frac{5}{2}$